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Analysis of mathematical argumentation of first-year students

Análisis de la argumentacion matematica de estudiantes de primer año

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Abstract

Fostering students at the beginning of university studies is an important issue not only in the international context but also in Chile. However, high dropout rates show that the transition from secondary school to university in mathematics is supposed to be difficult for many students. Mathematical argumentation is a core activity in mathematics and also very complex. Therefore it could be one of the main reasons for the difficulty at the secondary-tertiary transition in mathematics. This study analyzes the quality of students' argumentations, the proof scheme, and the formalism in students' answers of N = 86 first-year students in mathematics in order to better understand where the problems are in detail. The results show that the quality of argumentation, the use of appropriate arguments, and mathematical formalism highly varies in the two reported tasks: on the one side, 10% of students are able to prove one of the given geometrical theorems, on the other, 65% proved the other theorem correctly. This indicates that students are in a phase of developing their argumentative abilities. Consequently, the process of proving and constructing analytical arguments should be explicitly discussed in university lectures, so that students can improve their abilities for mathematical reasoning.

Keywords: higher education, secondary-tertiary transition, mathematical argumentation

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Resumen

Promover los estudiantes al inicio de sus estudios universitarios es un aspecto importante tanto en Chile como en el contexto internacional. Sin embargo, altos índices de abandono muestran que la transición educación media-educación superior es difícil para muchos alumnos, especialmente en matemáticas. La argumentación matemática es una actividad fundamental y compleja, la cual podría ser una de las principales razones de las dificultades que presentan los estudiantes en esta materia durante el proceso de transición. Este estudio analiza la calidad de las argumentaciones, los esquemas de prueba y el formalismo utilizado en las respuestas de N = 86 estudiantes de primer año, con el fin de entender mejor donde se ubica el problema. Los resultados muestran que la calidad de la argumentación, el uso de argumentos apropiados y los formalismos matemáticos tienen alta variación en las dos tareas reportadas: 10/% de los estudiantes probaron el primer teorema geométrico asignado y 65% probaron correctamente el segundo. Esto indica que los estudiantes están en una fase de desarrollo de sus habilidades argumentativas. Consecuentemente, el proceso de probar y construir argumentos analíticos debe ser explícitamente discutido en la Universidad, así los estudiantes podrán mejorar sus habilidades para el razonamiento matemático.

Palabras clave: educación superior, transición educación media – educación superior, argumentación matemática

Chilean situation

Access into Chilean higher education has shown an important improvement during the last years. According to the last report of the Ministry of Education (2016), the total undergraduate enrollment has increased by 57% between 2007 and 2016. This increase defines a new concern regarding the students´ transition from the secondary school to the tertiary education (Fonseca, 2011). The transition is seen as a difficulty resulting from several causes, the first being students' self-confidence, who think they are not prepared for higher education. Some of the other causes are associated to the university lack of strategies to tackle the challenge of social, cultural and cognitive diversity of new students (Herrera, 2011).

According to Herrera, González, Poblete, and Carrasco (2011), the failure represents a rupture created by the Chilean system between schools and universities, and it is of crucial importance to support the transition for ensuring the students´ continuity. Consequently, González (2011) states that it is important to do more research on this transitional process in order to make suggestions for policy makers to help young people to achieve their academic goals at higher educational levels.

The gap between school and university mathematics

According to the Centro de Medición de la Pontificia Universidad Católica de Chile (MIDE UC), students at secondary educational level show a high performance in problem analysis and conceptual understanding, particularly, in algebraic operations, properties of figures and geometric bodies, theorems of congruency and proportionality (Rodriguez et al., 2013). However, mathematical reasoning required to proof in geometry is a topic rarely taught in Chilean classes (Ministerio de Educación, 2015). Additionally, the absence of both formal argumentation and deductive reasoning in mathematical classes at school has called the attention of many researchers (Varas, Cubillos, & Jimenez, 2008).

After revising the preparatory programs that have been developed in different universities, Gutierrez et al. (2010) stablished that there is an important gap between the learning goals at secondary education and what students actually know when they are enrolled for tertiary education. Regarding the mathematical competence, Gutierrez et al. (2010) state that the required mathematics at the school level differs from the university, not only because of the topics, but also because a deeper understanding is needed. Learning mathematics at the secondary level includes mainly standardized processes, problem-solving strategies, and generalization of mathematical concepts; but students in higher educational levels are expected to have the ability to manipulate new objects with a better level of conceptual and abstract comprehension (Gutierrez et al., 2010).

Furthermore, according to Varas et al. (2008), the low level of awareness observed in Chile regarding the worth of mathematical reasoning should be faced at both, the curriculum level and the teachers' professional development level. Teachers from basic and middle school have shown similar deficiencies related to mathematical reasoning and theorem proving; they emphasize "calculation instead of analysis during mathematical classes" (Varas, 2008, p.53). As Radovic and Preiss (2010) stated, the instructional pattern in Chile emphasizes questions related to controlling class activities, more than enhancing the comprehension of mathematical concepts. Additionally, most of the questions have low cognitive challenge and reduce the student's opportunity to build knowledge while learning (Radovic & Preiss, 2010).

In 2006, 37% Chilean students who study science or mathematics, terminate their university studies or change their career during the first two years at university (Acuña, Makovec, & Mizala, 2010). Consequently, there are problems at the transition from school to university mathematics (Gallardo, Lorca, Morrás, & Vergara, 2014; Pérez, Castellanos, Díaz, González-Pienda, & Núñez, 2013).

Bridging the gap

In order to solve this problem, the Ministry of Education has suggested a pedagogical material in which it is important for learners to understand the fact that in mathematics, all propositions can and should be proved. In this material the main goal is to develop the analytical and deductive thinking before students starting with the proof of theorems. In this way, students should firstly understand the need to proof; then, the concepts of hypothesis and thesis as well the concepts of mathematical proof (Ministry of Education, 2015). The "Didactic guidelines and guides for the student", created by the Ministry of Education, provides examples of proof using both narrative language and mathematical writing, and it is recommended that teachers to use these "according to the student needs" (p.58).

In addition, during the last years the Ministry of Education in Chile has generated different strategies for founding programs aiming to support first-year students in higher education institutions. As an example, the preparatory program "Introduccion a la matemática universitaria" (PIMU) has been one of the most important activities developed at tertiary level (Portales ,2015). This course is conducted before the classes at university begin, and the goal is to help learners to approve mathematics courses at the first year.

Aiming to determine the effectiveness of the PIMU, Portales (2015) analyzed the students' performance in mathematical courses offered during the first year. The study found the program has a poor impact on students with a low performance in the diagnostic test; it has a positive contribution in groups with average result, and low contribution for groups with high performance. The study also identified that the mathematical knowledge differ between students participating in the course, and preparatory program is not enough for covering all the deficiencies that students bring from the school (Portales, 2015).

Mathematical argumentation

At the secondary-tertiary transition students have to overcome several gaps in order to successfully manage their start at the university level in mathematics. As mathematical argumentation is a key competence in mathematics (CCSSI, 2010; Ministerio de Educación, 2011) we will emphasize the process of mathematical argumentation as an important competence that should be fostered during secondary school and at university level. In this article, mathematical argumentation means proving mathematical theorems. However, there is also research which analyzes students' argumentation in the classroom which emphasizes the dialogue between the learners, or the learners and the teacher (e. g. Pedemonte, 2007; Solar, Giménez, & Piquet, 2012; Solar & Deulofeu, 2104). This research area instead focuses on the structure of an argument and is mostly based on Toulmin's model of arguments (Toulmin, 2003).

While there are studies from different countries which analyze mathematical argumentation of students (e. g. Heinze, Cheng, Ufer, Lin, & Reiss, 2008; Heinze & Reiss, 2007; Reiss , Heinze, Renkl, & Groß, 2006; Healy & Hoyles, 2000), there are only few studies exploring this topic at the tertiary level. Mathematical argumentation is essential due to its important role in the development of mathematical theories. It is, therefore, a core part of academic mathematics and plays an essential role in mathematics studies at university. The typical structure of a mathematical theory is *definition – theorem – proof*. It is also the typical structure of mathematical lectures at university that further demonstrates the importance of proofs and proving. Proofs are not only a fundamental part of mathematics but also assist with better understanding mathematical content (Hanna, 1995, 1997; Hanna & de Villiers, 2008). They explain why theorems are true and show relationships between mathematical concepts. Boero (1999) describes the procedure of how experts in mathematics usually develop formal proofs by means of a model which includes the following six steps:

> I) production of a conjecture […];
> II) formulation of the statement according to shared textual conventions […];
> III) exploration of the content (and limits of validity) of the conjecture; […] elaborations about the links between hypotheses and thesis; identification of appropriate arguments for validation […];
> IV) selection and enchaining of coherent, theoretical arguments into a deductive chain […];
> V) organization of the enchained arguments into a proof that is acceptable according to current mathematical standards […];
> VI) approaching a formal proof […]
> (Boero, 1999, p. 2)

At the beginning of the proving process there is a conjecture which should be proved. Then, experts formulate possible hypotheses and reflect the theorem and its meaning. Furthermore, they explore the mathematical problem and connect it with related concepts of the same mathematical context. Also they collect plausible arguments for proving. In the fourth step the experts select relevant arguments and order them deductively. Finally, they approach a formal proof.

How the proving process at secondary school can be characterized describes Brunner (2014). She presents a model which shows the development of a formal proof. At the lower level, there is the experimental proof which is connected to concrete examples. By choosing appropriate arguments and generalizations---firstly from a visual perspective---students are able to develop a formal proof which is based on deduction.

Both models demonstrate that a formal proof is the result of a process including many steps. However in university lectures, mathematical proofs are mostly presented in their final version, the process of proving is therefore, not always evident for the students. The fact that students typically do not see how a proof is created, could be a reason why they have problems in learning and practicing argumentation. Mathematical argumentation is not only challenging for students as they do not usually see the proving process, but also it is one of the most complex activities in mathematics. For proving correctly, students should have abilities to solve advanced mathematical problems. They additionally should know sophisticated strategies and the structure of a proof (Reiss & Ufer, 2009). If students, for example, should prove why the sum of all angles inside of a triangle equals 180°, they not only have to know and understand the content but also should know that they need arguments which are ordered deductively. They furthermore, need general problem-solving abilities which allow them to develop an appropriate solution.

Consequently, many first-year students do not know how to start a mathematical proof (Moore, 1994). The results of other studies with students at the university level also illustrate that the quality of argumentations are rather low (Nagel & Reiss, 2016). This explains why mathematical argumentation is one of the most challenging mathematical activities at the beginning of university studies (Engelbrecht, 2010).

Proof schemes

Not only the quality of students' proofs is possibly low, but also the arguments they use for proving might not be appropriate for mathematics at university level. Harel and Sowder (1998) analyze arguments of students at the beginning of their university studies. They distinguish between three main schemes of reasoning:

- External conviction
- Empirical arguments
- Analytical arguments

Students use external arguments if they are convinced of a theorem being true but cannot give a mathematical argument for that. They rather refer to external authorities, for example, textbooks or teachers. Empirical arguments are mainly based on examples and have an inductive character. Analytical arguments are based on logical deduction and represent the correct mathematical way of proving a theorem.

Also Recio and Godino (2001) analyzed the types of arguments of students at university. They connect the different types of arguments to the contexts "daily life", "experimental sciences", "professional mathematics", and "logic and foundations of mathematics" (Recio & Godino, 2001, p. 83). They found four main types of arguments (Recio & Godino, 2001, p. 97):

- Personal explanatory argumentation schemes
- Empirical-inductive proof schemes
- Informal deductive schemes
- Formal deductive proof schemes

A comparison between the schemes of Harel and Sowder (1998) and Recio and Godino (2001) shows that both mainly differentiate between inductive and deductive arguments. Similarly, an exploratory study of Martin and Harel (1989) with first-year students at university could also find these two schemes in general.

Students at the secondary and tertiary level do often not use deductive arguments for proving. Studies with students of secondary schools show that they instead often use empirical arguments (e. g. Heinze & Reiss, 2007; Healy & Hoyles, 2000). Even at the university level, students frequently use empirical arguments for proving a mathematical theorem (Knuth, 2002; Martin & Harel, 1989). However, results of a different study indicate that most of the first-year students reason analytically (Nagel & Reiss, 2016).

Formalism in mathematics

The mathematical content becomes more complex at university level in comparison to content at school and it is presented more formally. A reason for that is that students should be introduced into the formal mathematical language which is essential for academic mathematics. In the first year at university, students are not yet used to formal language and thus, this can cause problems in learning and understanding mathematical content at university level properly. If mathematical content is presented formally, it is not easy to create appropriate visualizations and generate examples of the mathematical concepts which would be necessary for understanding them (Tall & Vinner, 1981).

Studies with students at school show that they have problems in writing their answers in an adequate mathematical language (Hoyles, Newman, & Noss, 2001; Fischer, Heinze, & Wagner, 2009). For solving mathematical problems which require mathematical argumentation, formally presented content can increase the difficulties at the beginning of university studies. Lakatos (1979), for instance, criticized the emphasis on formalism in mathematics as it does not help in developing mathematical theories. He states that the logic should be the focus. Studies with students at school demonstrate that students evaluate rather formal proof as being correct than not formal proofs (Wittmann & Müller, 1988). This shows that strict formalism tend to focus formal aspects of a proof instead of the content.

For analyzing the ability of first-year students to reason mathematically we need a special instrument. As mathematical argumentation requires several abilities, the instrument should focus on few important aspects. One of those should be the quality of students' argumentations. Furthermore, the type of argument students are using should be examined to better understand where the difficulties in reasoning mathematically are in detail. Thirdly, it should measure if students are able to use formal mathematical language when proving.

Research Questions

For analyzing mathematical argumentation of first-year students at university, the first research question is:

1. How is the relative frequency of correct argumentations?

As studies with students at the secondary level as well as studies with first-year students in mathematics show that the quality of argumentation is rather low (Nagel & Reiss, 2016; Reiss et al., 2006; Heinze & Reiss, 2007; Knuth, 2002; Healy & Hoyles, 2000; Martin & Harel, 1989), we assume that students have difficulties in reason mathematically and, therefore, the argumentation quality is probably low.

The second research question analyzes the proof scheme students are using:

2. Do students use empirical arguments for proving?

Studies with students at secondary school level show (e. g. Reiss et al., 2006; Heinze & Reiss, 2007; Healy & Hoyles, 2000), that they mainly use empirical arguments which are based on examples. A study with students at the tertiary level however, indicates that they in general reason analytically (Nagel & Reiss, 2016). As the empirical results are not consistent, we assume that they use empirical or analytical arguments.

The third research question is about the use of the formal mathematical language when proving a theorem:

3. Do students use formal mathematical language?

Studies of Hoyles et al. (2001) or Fischer, Heinze, and Wagner (2009) demonstrate that students have problems in expressing their answers in a formal mathematical way. Because of that we suppose that students express their answers not formally but rather narratively which means with words.

Method

The instrument measures three aspects: quality of argumentation, type of argument, and use of formal mathematical language. For detailed information about the proving process of the students we created three open-ended questions which illustrate the thinking process and development of students' argumentations better than multiple-choice items. Other researchers who analyzed mathematical argumentation also used open-ended questions (e. g. Nagel & Reiss, 2016; Reiss et al., 2006).

The content of the tasks is related to geometrical content which is part of the school curriculum (Common Core State Standards Initiative, 2010; Ministerio de Educación, 2011) and is considered to be essential for school geometry. This way, we are able to separate the ability of mathematical reasoning and conceptual knowledge. If students are not able to prove one of the given theorems correctly, we can probably assume that at least the mathematical content was familiar to them and that there must be difficulties in developing mathematical arguments. Another reason for choosing geometrical content is that it allows graphical solutions which reveal the proving process more clearly than, for instance, algebraic transformations of mathematical terms.

Items

The instrument includes three theorems that students should prove. They have 30 minutes to finish the test. In the first task the participants should prove why the three bisectors of a triangle intersect. In the second task they should prove why the Thales' theorem is true. In the third task they should prove the Pythagorean theorem. This article only reports the results of the first and the second task.

To solve the first question in which the participants have to prove the existence of the intersection of the three bisectors of a triangle, it is necessary to know that all points on the bisectors have the same distance to the two vertices of this side. Furthermore, it can be argued that the intersection of two bisectors is equidistant from all three vertices of the triangle. Finally, the third bisector must pass through the intersection, regarding the definition of a bisector of a triangle's side.

In the second task the participants have to prove the Thales' theorem. The task requires appropriate angle considerations to show that the angle at the corner of C of the triangle ABC is equal to 90 °. As a first step the triangle must be divided into two isosceles triangles with equal base angles. Furthermore, in regard to the sum of all angles in a triangle which measures 180°, it can be argued that the angle at the vertex C equals 90°.

Evaluation of the instrument

We also analyzed the quality criteria of the measuring instrument. Objectivity is given, if the results of a test are independent from external influences (e. g. Sedlmeier & Renkewitz, 2013). The test was carried out in two comparable exercise groups. All participants have received the same instructions and the test was taking place at the same time. Furthermore, the test was analyzed by two raters who show a good agreement in their coding results. Therefore, we assume that the instrument was objective.

The reliability of an instrument usually is examined via the Cronbach's Alpha coefficient which analyzes the internal consistence of the items (Cronbach, 1951). As only two of the three items were analyzed yet, the scale of the coefficient only would include two items. Therefore we assume that the coefficient is rather low as it is dependent to the number of items. However, we calculated the Cronbach's Alpha coefficient which was $r = 0.291$. Due to the low number of items we can interpret this value as satisfying.

Thirdly, we controlled the test validity of the content by consulting experts in mathematics education and mathematics who selected the content of the items carefully.

Coding

The tasks were evaluated by means of a coding scheme, which corresponds to the proof schemes of Harel and Sowder (1998). It contains the variables "quality of argumentation", "proof scheme" and "type of answer" (see Table 1).

Table 1

Coding variables

Quality of argumentation	Other – no approach – approach / idea – sub-steps of a proof – correct proof
Proof schemes	No arguments – external arguments – empirical arguments – analytical arguments
Formalism	No arguments – narrative answer – narrative and formal answer – formal answer

The two tasks were coded by two independent raters in order to control the interrater reliability and the objectivity. The interrater reliability was calculated with Cohen's kappa, which are presented in Table 2. The coefficients indicate a high degree of agreement between the two raters (Cohen, 1960).

Table 2

Interrater-reliability of both tasks

Coding variables	Cohen's kappa	
	Item: Intersection of the three bisectors of a triangle's side	Item: Thales' theorem
Quality of argumentation	0.76	1.0
Proof schemes	0.74	1.0
Formalism	0.73	0.79

Data collection

The sample includes N = 86 first-year students of the Pontíficia Universidad Católica de Chile, Santiago de Chile. The participants study mathematics, mathematics education, physics, or physics education. They take part in the same mathematics lectures. The cross-sectional study was carried out in the first semester during a mathematics exercise lesson.

Results

Table 3 shows the relative frequency of all variables in both tasks.

Table 3

Relative frequency of all variables in both tasks

Variables	levels	Item: Intersection of the three bisectors of a triangle's side	Item: Thales' theorem
Quality of argumentation	other	0.37	0.12
	No approach	0.33	0.07
	Approach / ideas	0.12	0.02
	Sub-steps of a proof	0.09	0.14
	Correct proof	0.10	0.65
Proof schemes	No arguments	0.65	0.20
	External arguments	0.14	0.02
	Empirical arguments	0.02	0.04
	Analytical arguments	0.19	0.74
Formalism	No arguments	0.56	0.15
	Narrative answer	0.22	0.16
	Narrative and formal answer	0.20	0.58
	Formal answer	0.02	0.11

More than a third (37%) of the students' answers is coded as "other". These participants mistake the bisector of a triangle's side with the height of a triangle or the medians of the triangle. The quality of argumentation is low for the item in which the participants have to prove why the three bisectors of a triangle's side intersect. 33% do not give any arguments for explaining the theorem mathematically. However, only 12% of the students do not use any arguments for proving the Thales' theorem. The majority of the participants (65%) prove Thales' theorem correctly, whereas only 10% are able to prove the first task correctly. Furthermore, 14% of the students generate sub-steps of the proof regarding the Thales' theorem.

In terms of the used proof scheme, the results indicate that many students do not reason with arguments at all: in the tasks in which they have to prove the intersection of the bisectors, 65% of them do not use mathematical arguments. In the second task in which they should explain why the Thales' theorem is true, 20% of the students do not use arguments. Very few students (2.0% and 4.0%) reason empirically. In the task in which they have to prove the Thales' theorem, 74% argued analytically. However, only 19% use analytical arguments when proving the theorem of the intersection of the bisectors.

With regard to the formalism, the answers of the first task (intersection of the three bisectors) are mainly written with words which means narratively, whereas the answers of the second task (Thalens' theorem) were considered as more formal. 56% of the students cannot find any arguments for proving that the three bisectors of a triangle's side intersect. If students use arguments in that task, they are mainly narrative (22%). In comparison, 58% present their solution in a mixed way which is coded as "narrative and formal". A formal argumentation is identified in 2.0% of the answers in the first task and in 11% of the answers in the second task. Nevertheless, the relative frequency of the narrative answers is higher than the relative frequency of the formal arguments in both tasks.

Discussion

At the beginning of university studies, there are many difficulties for students indicated by high dropout rates in mathematics (Acuña et al., 2010). Especially the content and mathematical argumentation is supposed to be challenging for first-year students (Hoyles et al., 2001, Engelbrecht, 2010). As mathematical argumentation is one of the most important but also complex mathematical activities (Boero, 1999; Brunner, 2014), we focus on three main aspects of mathematical argumentation of students in their first year at university. These are: quality of argumentation, type of arguments, and formalism in their answers. We constructed a measuring instrument, which give insights into the process of proving. Consequently, we use three open-ended tasks in which first-year students have to prove given theorems. In this article, we only report the results of two of the three items as the analysis is not yet finished. The theorems of the tasks are related to geometrical content that the students already should know from secondary school. This way, we assume that students should be familiar with the content and the difficulty for solving the tasks are mainly related to their argumentative abilities.

The results of the analysis indicate that the content of the tasks plays an enormous role for proving correctly. A majority of the first-year students (65%) are able to prove the Thales' theorem correctly, whereas only 10% are able to do so in the task which requires a proof of the intersection of the three bisector of a triangle's side. The arguments that should be used for proving the theorem of the intersection of the bisectors are probably more complex and abstract so that the students have more difficulties in proving it. One of the arguments is based on the distance between the vertices and the bisector. The arguments for proving the Thales' theorem correctly mainly refer to elementary geometric content, for example angles. It seems that developing and selecting the right arguments are difficult steps with respect to Boero's (1999) model. Also exploring the content of the mathematical theorem seems to be complicated and is deeply connected with the mathematical concepts of the theorem. 33% of the students are not able to give any mathematical arguments for proving that the three bisectors of a triangle's side intersect. This confirms that in general, students at the beginning of university studies have great difficulties in developing mathematical arguments.

The results regarding the proof schemes are not consistent. Whereas the majority of students (74%) reasons analytically in the task in which they have to prove the Thales' theorem, only few students (19%) use analytical arguments in the other task. That indicates that many students are generally able to order their arguments deductively. However, they have not yet the ability to develop deductive arguments in both tasks. They can obviously not transfer their knowledge about the correct structure of a mathematical proof to other tasks. This confirms the inconsistent results of other studies which analyze mathematical argumentation of first-year students. Some studies document the frequent use of empirical arguments (Knuth, 2002; Martin & Harel, 1989), whereas other studies indicate that first-year students mostly use analytical arguments (Nagel & Reiss, 2016). A reason for these results could be that at the transition from school to university mathematics, students begin to learn how the structure of mathematical proofs is. As proofs are usually presented in their final version in university lectures, the development of this knowledge is not constant. There are obviously some tasks which first-year students can prove deductively but there also are tasks which they cannot explain with mathematical and deductive arguments.

By analyzing the formalism of the students' answers we find that in the task in which students should prove the Thales' theorem, the majority (58%) used a mixture of narrative expressions and formalism. In the other task which requires a proof of the intersection of the three bisectors of a triangle's side, 20% used a mixture of words and formalism. 22% of them only used words for proving why the three bisectors intersect. As expressing answers with formalism is more difficult for students (e. g. Wittmann & Müller, 1988), it is not surprising that they do not use formalism for proving the theorem that the three bisectors of a triangle's side intersect. The answers of the task referring to the Thales' theorem, which is supposed to be easier for the students, are expressed more formally. It seems that if students can order their arguments in a deductive chain, they are also able to express them formally.

The study gives insights into the proving process of students at the beginning of their university mathematics studies. The geometrical content and the item format allow analyses of the quality of argumentations, the proof schemes and the formalism of students' answers. The quality criteria---objectivity and test validity---of the instrument are good, whereas the Cronbach's Alpha coefficient for the internal reliability was rather low due to the fact that the scale only includes two items. We analyzed students' argumentation in mathematics with only two tasks. Therefore, the results should be interpreted with caution. For general conclusions, more research is needed. Nevertheless, the results illustrate several problems that students have when proving a mathematical theorem at the beginning of their mathematics studies. Mathematical argumentation is a complex process which requires different and advanced abilities (e. g. Reiss & Ufer, 2009). This study points out detailed information about where difficulties in mathematical argumentation possibly are. One of the difficulties is using analytical arguments for proving which are connected deductively. The results show that the students obviously are in a phase in which they start developing their mathematical argumentation abilities. However, in the first task the argumentation quality was rather low and they used mainly no arguments for proving. In the second task they perform well in both variables. The research indicates that students in their first year at university need more learning environments in which they can foster their abilities in mathematical reasoning. Therefore, the results of this study could be a basis for a generating support for learning how to prove mathematically which is adapted to the abilities of first-year students.

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